2024-2025
Fall Semester

Course of Power System Analysis

Short-Circuit Current Interruption

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Short-circuit current interruption

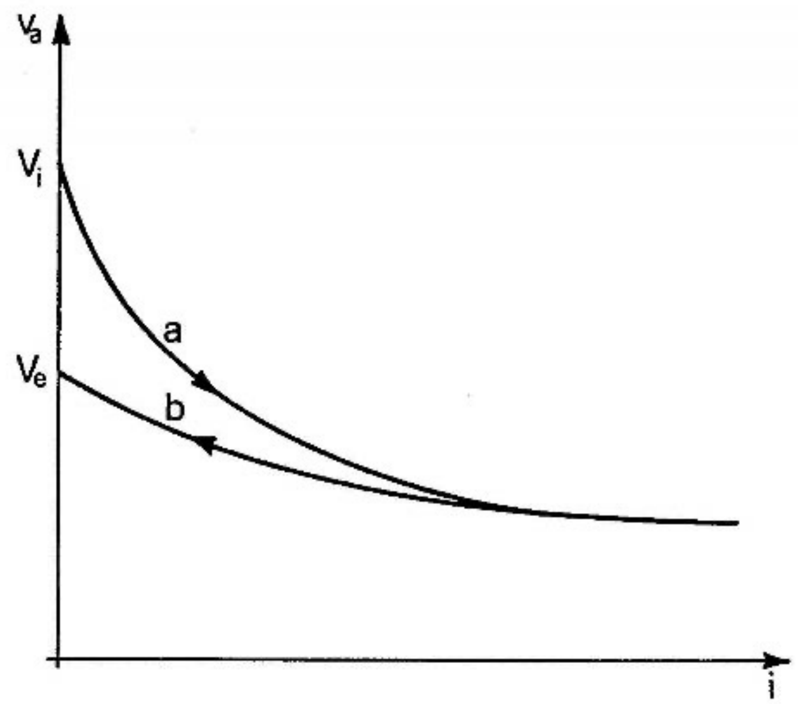
As soon as an electric current is interrupted, a **highly ionized zone** is created at the point of interruption, at a very high temperature (around 5000 K). Normally, this zone will generate an **electric arc**.

IMPORTANT: the characteristics of this arc are used by circuit breakers to help cut off the electric current while controlling the arc itself.

Short-circuit current interruption

$V - I$ characteristic of the arc for DC

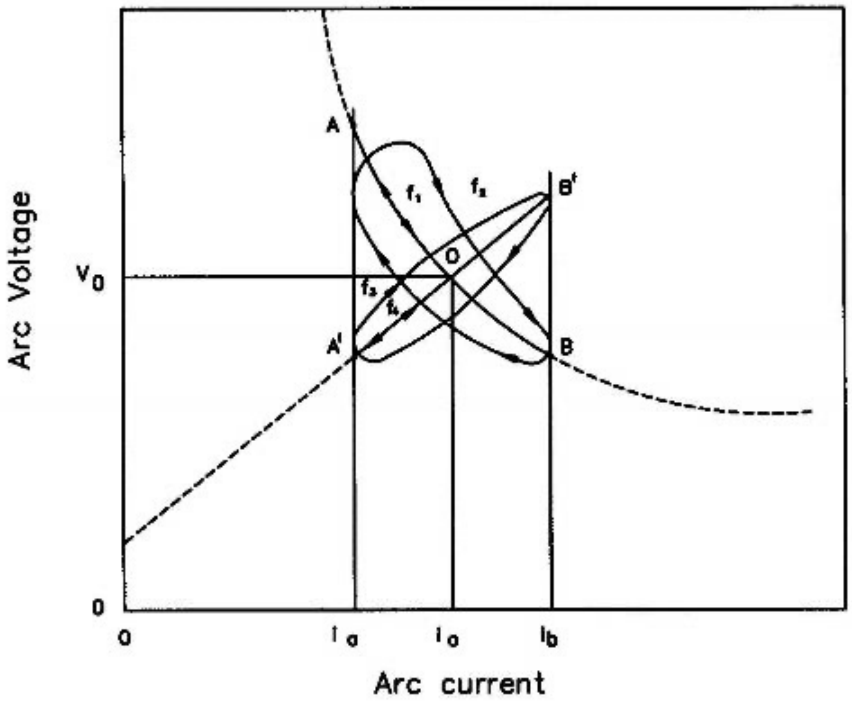
- Characteristic obtained by **several states of equilibrium**;
- It is **non-linear and opposite to an ohmic characteristic** (if the current increases, the ionization phenomena of the gas and electrodes become more significant, and the arc voltage drop decreases);
- **hysteresis**: for a current drop, the voltage of the electric arc does not reach the same values as if the current were to be increased.



Short-circuit current interruption

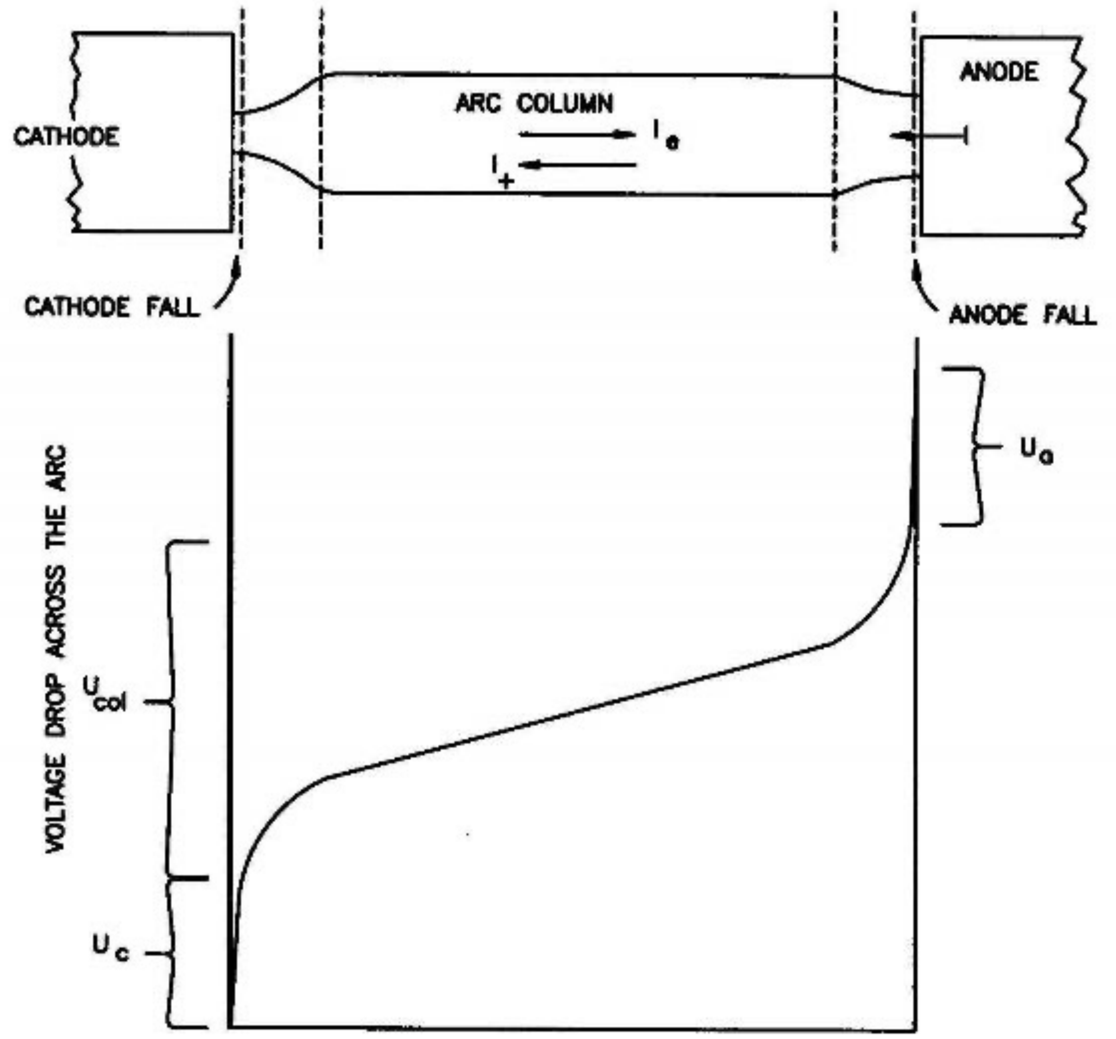
$V - I$ characteristic of the arc for AC

- For AC, the characteristic depends on the **supply frequency.**
- **at low frequencies** (f_1) the arc conductance can follow current variations, and the V-I characteristic of the arc **coincides with that of DC.**
- **at high frequencies** (f_4) we can observe that **arc conductance is unable to follow current variations** → the *V-I* characteristic is **ohmic.**



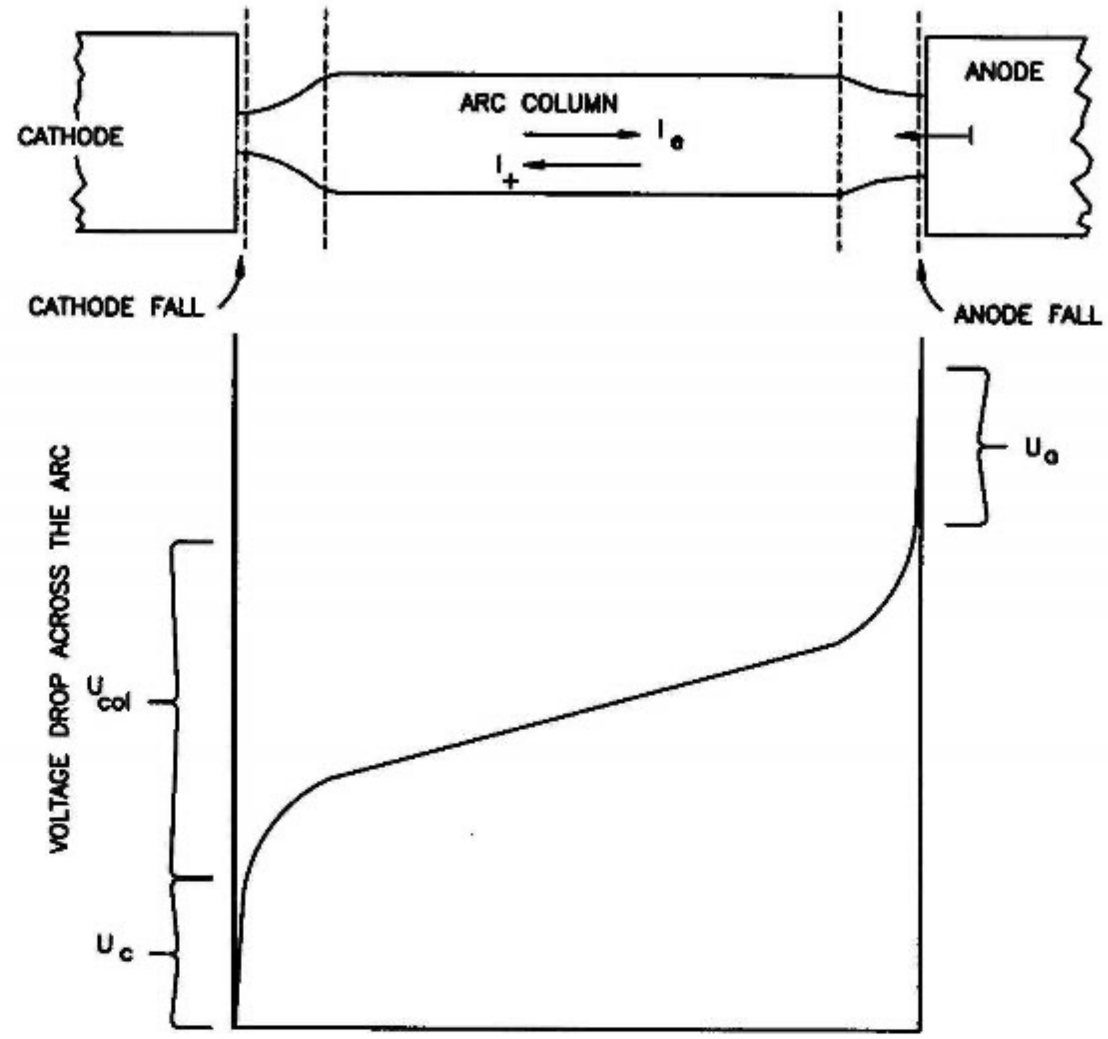
Short-circuit current interruption

The voltage of the arc v_a is mostly **concentrated in the zones of the cathode and anode** electrodes, while voltages in the arc column zone are low.



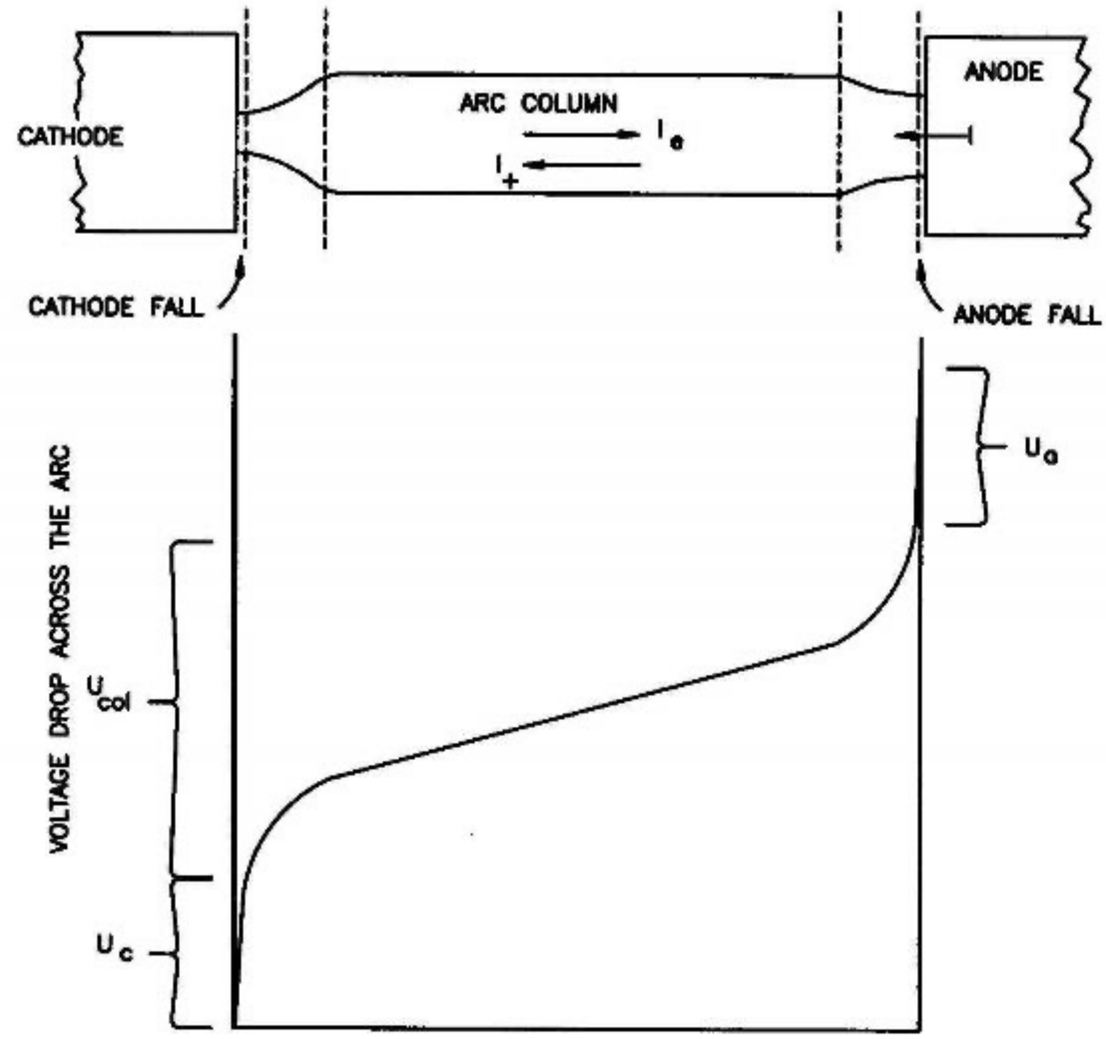
Short-circuit current interruption

Observation #1: the voltage of the arc **does not depend on the supply voltage** of the circuit, but rather **depends on the power necessary to supply the arc.**



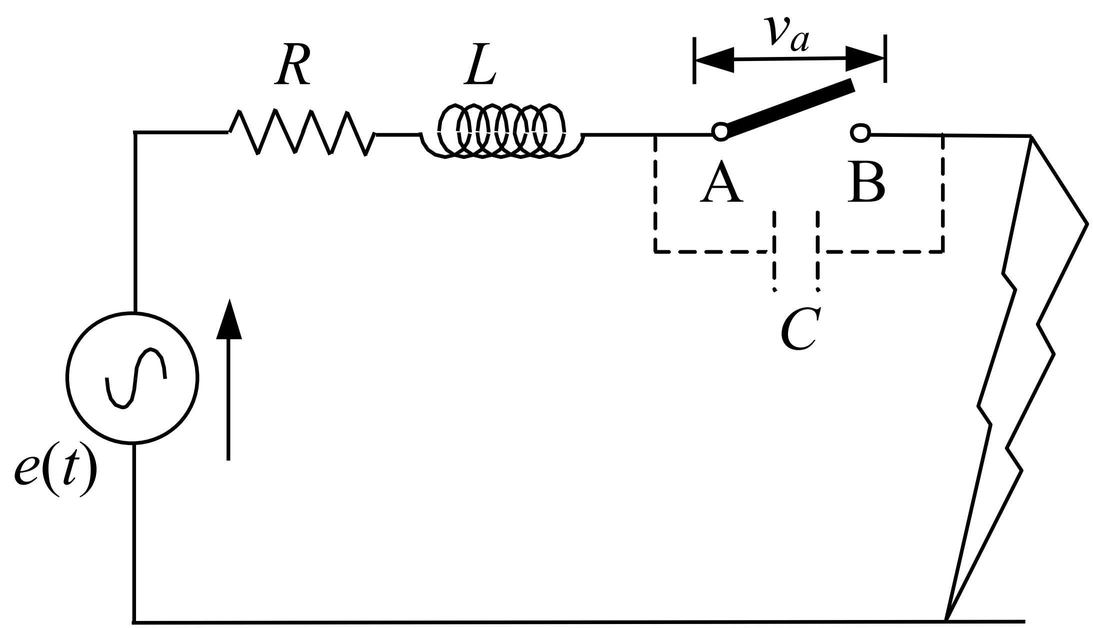
Short-circuit current interruption

Observation #2: if the arc is **cooled down, the arc voltage drop must increase**, as more power is needed to sustain the arc itself.



Short-circuit current interruption

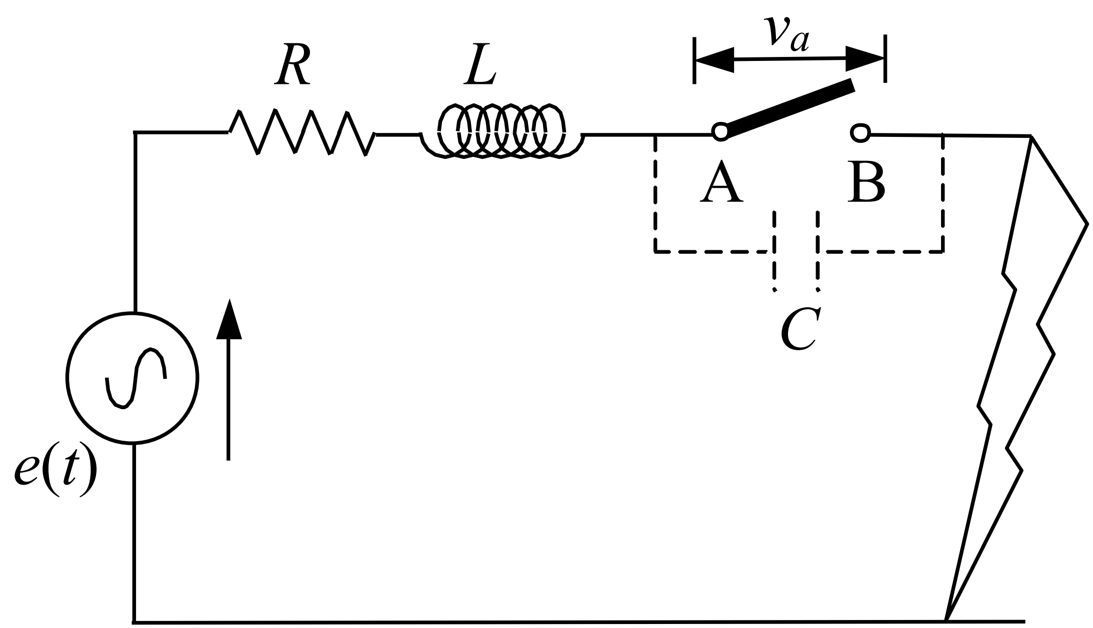
Equivalent circuit for analyzing transient phenomena which occur **at the point of short-circuit current interruption** by a circuit breaker.



Short-circuit current interruption

Hypotheses:

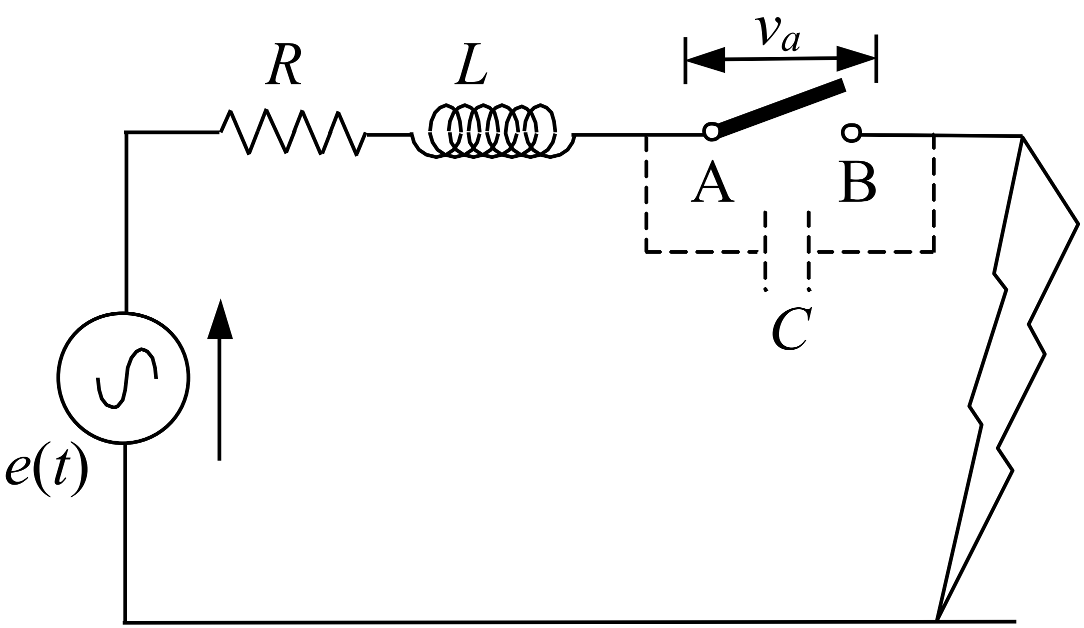
- the network upstream of the circuit breaker is modeled by the Thévenin equivalent circuit;
- between points A and B, which represent the poles of one phase of the circuit breaker, arc voltage v_a will be maintained until the arc interruption is completed.



Short-circuit current interruption

Hypotheses:

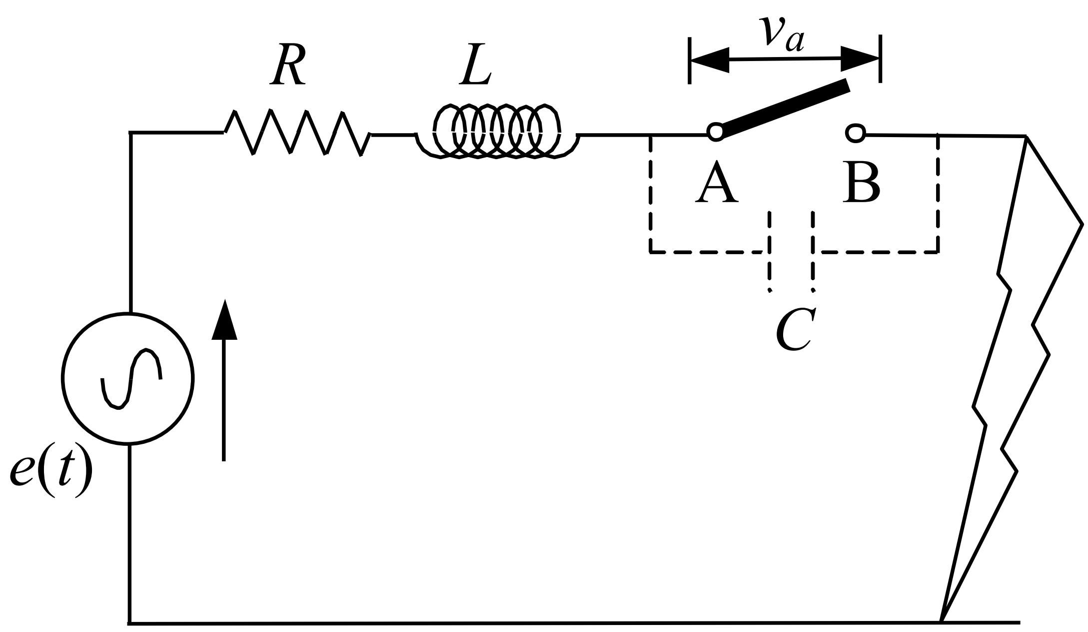
- as soon as the arc is extinguished, a voltage $u(t)$ will be present between points A and B, and an electric current will flow due to the stray capacitance between the open contacts of the circuit breaker.



Short-circuit current interruption

Equation of the equivalent circuit at the **instant prior to the current interruption**

$$e(t) = v_a(t) + Ri(t) + L\frac{di(t)}{dt}$$



Short-circuit current interruption

Important observation: for the same current value, the voltage drop v_a depends **only on geometric factors and arc temperature**.

As long as the arc voltage does not depend on the value of the supply voltage, it can be said that at low voltage, it is possible to manage the phenomenon such that the arc voltage $v_a(t)$ **has a comparable value to the voltage $e(t)$ of the studied circuit**. This observation is extremely important for the design of **low-voltage circuit breakers**.

Short-circuit current interruption

If the circuit-breaker is built in such a way that it is possible to have a value of $v_a(t)$ **comparable to (or higher than) the voltage imposed by the generator, we have**

$$e(t) - v_a(t) < 0$$

Therefore, by substitution

$$Ri(t) + L\frac{di(t)}{dt} < 0$$

given that the short-circuit resistance of the network upstream of the circuit breaker is much lower than the short-circuit reactance of the same network (R << L), we have

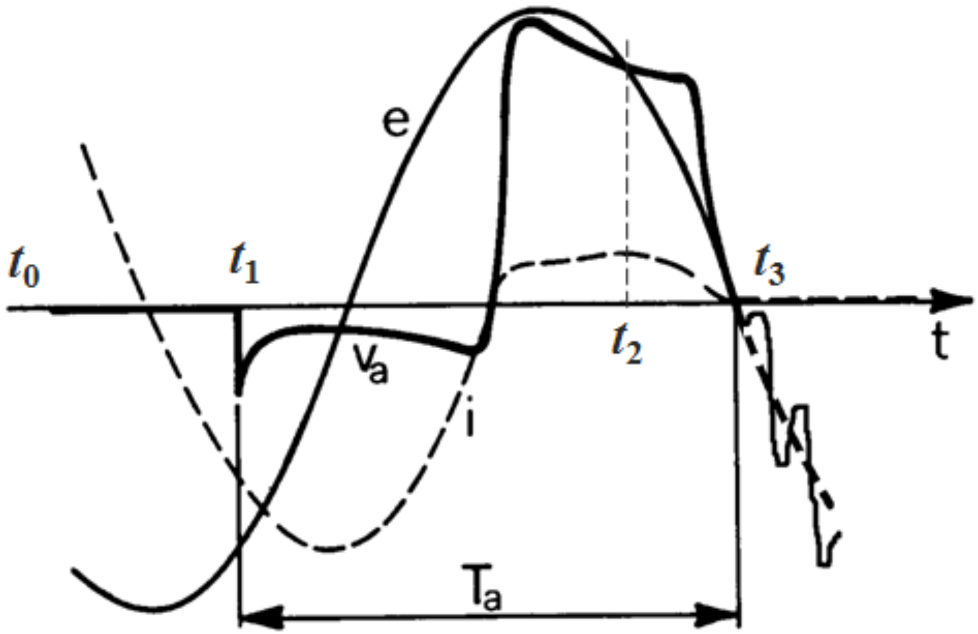
$$\frac{di(t)}{dt} < 0$$

Short-circuit current interruption

We obtain a **distortion of the current waveform with an anticipated passage through zero**. If the current has a value of zero, the arc is extinguished, and if the **circuit breaker poles are far enough apart and the ionized gas around them has cooled**, then the arc will not re-ignite and the interruption operation is completed.

Short-circuit current interruption

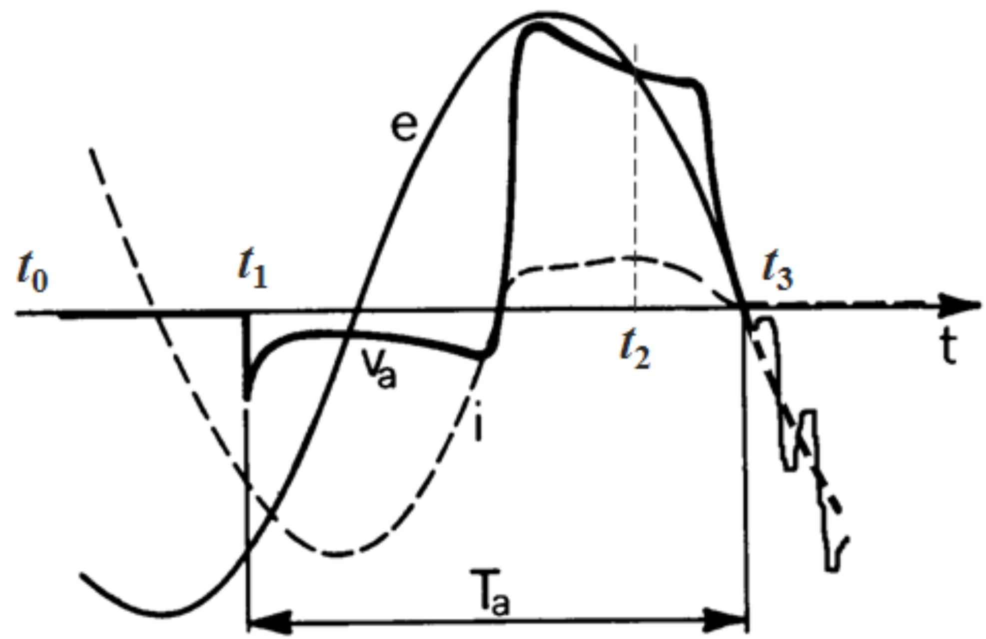
- t_0: start of the short-circuit;
- t_1-t_0: time necessary for the protections to react;
- t_1: separation of contacts and start of the arc phase;
- t_2: instant during which $v_a(t) > e(t)$ and the current is forced downward;
- t_3: instant at which zero current is achieved and the arc is extinguished.



Short-circuit current interruption

Observation: if the arc voltage is **comparable to the supply voltage**, the arc is extinguished **each time the voltage crosses zero.**

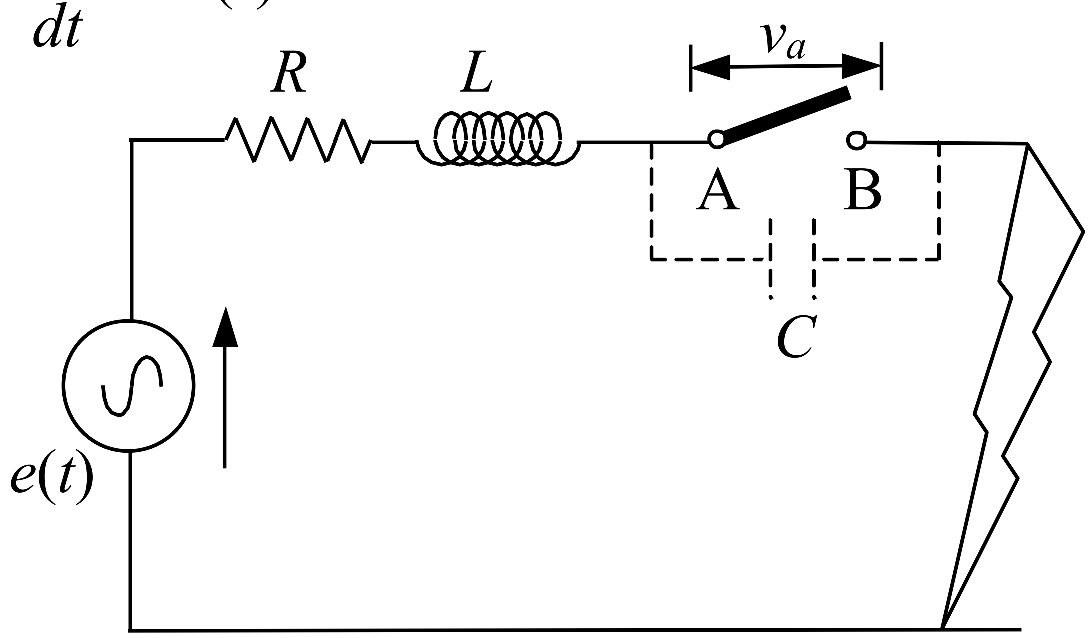
This condition is favorable because the value of the voltage $u(t)$ across points A and B just after the arc is extinguished is very close to the imposed voltage $e(t)$.



Short-circuit current interruption

Equation of the equivalent circuit at the **instant after current interruption**

$$e(t) = E_M sin(\omega t) = Ri(t) + L\frac{di(t)}{dt} + u(t)$$



The voltage $u(t)$ is between the points A and B, thus:

$$i = C\frac{du(t)}{dt}$$

Short-circuit current interruption

Therefore, by substitution:

$$E_M sin(\omega t) = RC\frac{du(t)}{dt} + LC\frac{d^2u(t)}{dt^2} + u(t)$$

We have taken as the initial instant of the transient the **instant at which the arc is extinguished** (at which point the voltage $e(t)$ passes through zero), and so we can write

$$e(t) = E_M sin(\omega t)$$

In the differential equation, the resistance R can be neglected, as R << L, and above all because we are interested in the $u(t)$ curve following the current interruption, i.e. before the damping of oscillations due to energy dissipation on resistance R takes place.

Short-circuit current interruption

With this simplification:

$$\frac{E_M}{LC} sin(\omega t) = \frac{d^2 u(t)}{dt^2} + \frac{u(t)}{LC}$$

If $\mu_0 = \frac{1}{\sqrt{LC}}$, where μ_0 is the circuit's natural pulse:

$$\mu_0^2 E_M sin(\omega t) = \frac{d^2 u(t)}{dt^2} + \mu_0^2 u(t)$$

The solution is as follows:

$$u(t) = E_M sin(\omega t) + A sin(\mu_0 t) + B cos(\mu_0 t)$$

Short-circuit current interruption

$$u(t) = E_M sin(\omega t) + Asin(\mu_0 t) + Bcos(\mu_0 t)$$

For t=0 $u(0)$=0 → B=0

We also have that

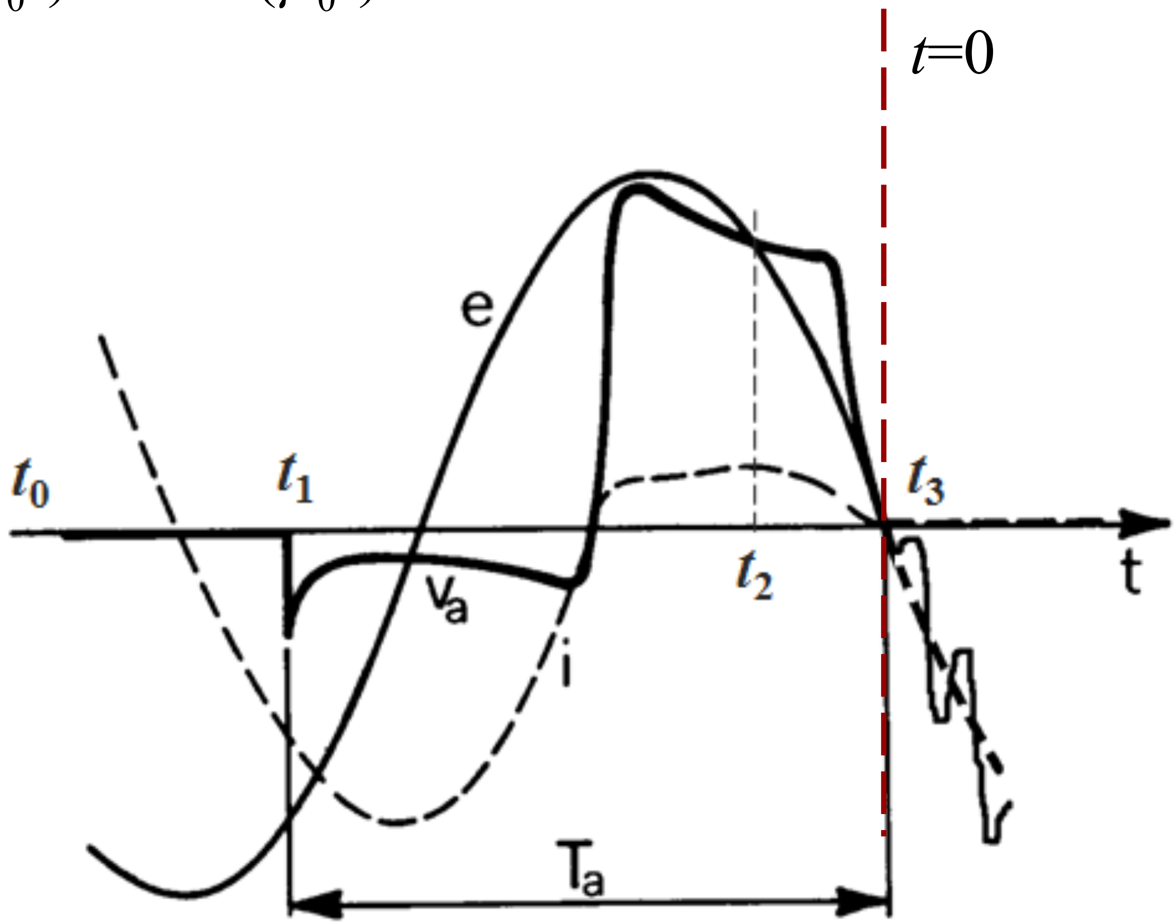
$$i(0) = C \frac{du(t)}{dt}\bigg|_{t=0} = 0$$

Therefore

$$A = -\frac{E_M \omega}{\mu_0}$$

and the solution is

$$u(t) = E_M sin(\omega t) - E_M \frac{\omega}{\mu_0} sin(\mu_0 t)$$



Short-circuit current interruption

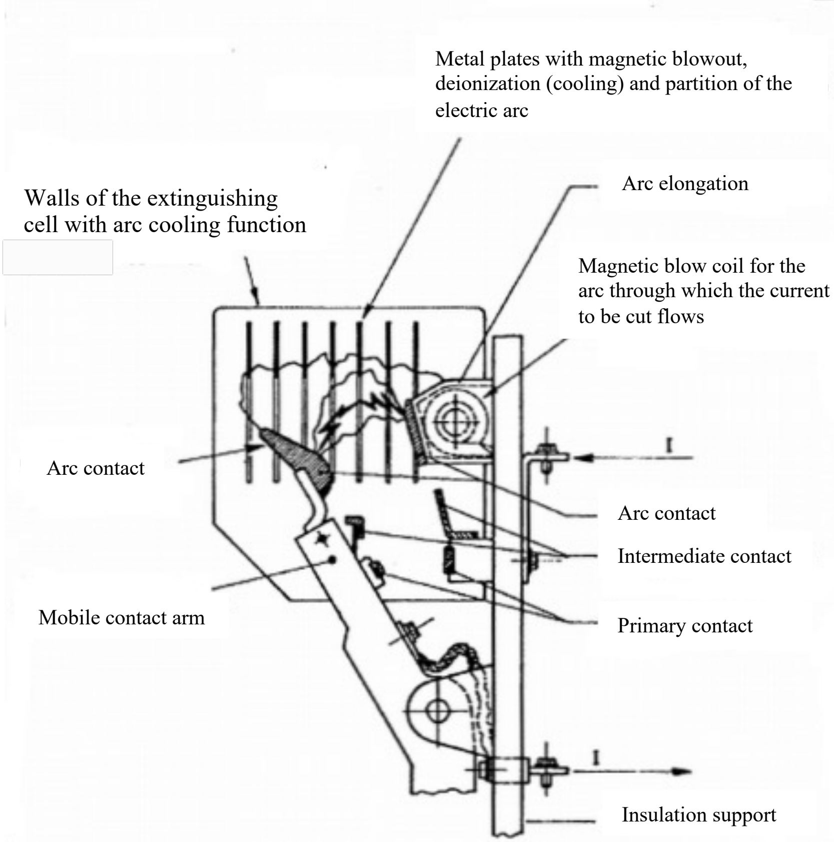
For **typical values of inductance and capacitance** found in electrical circuits, we obtain
$\mu_0 >> \omega$.

Consequently, the term oscillating at the natural frequency μ_0 is characterized by a **much smaller amplitude than that of the term at the nominal frequency** ω, therefore:

$$u(t) = E_M sin(\omega t) - E_M \frac{\omega}{\mu_0} sin(\mu_0 t) \cong E_M sin(\omega t) = e(t)$$

Short-circuit current interruption

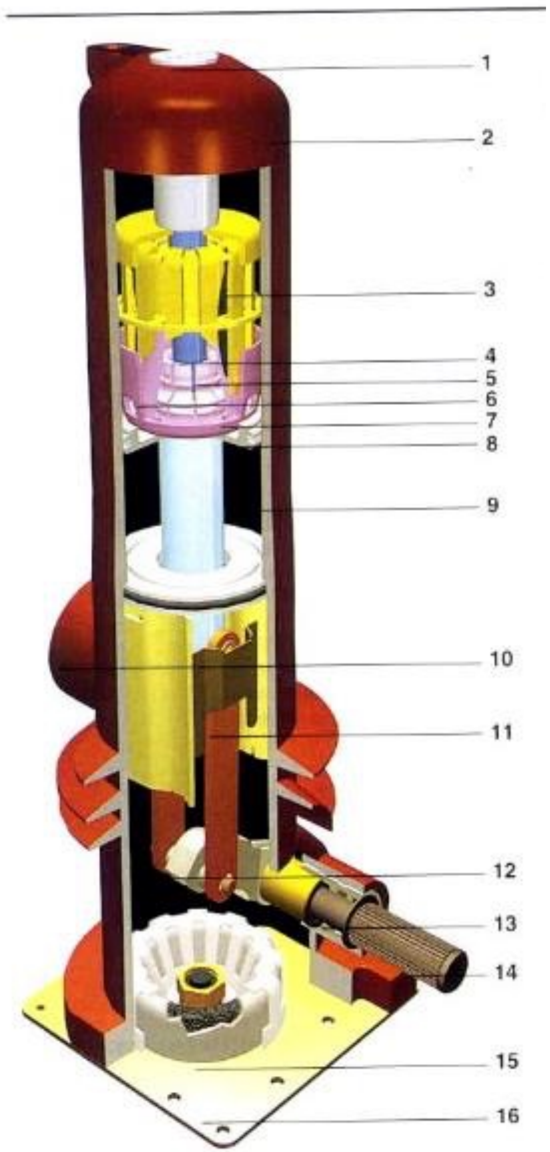
Recall: these equations are valid only if $v_a > e$.
This condition is met for **circuit breakers in low-voltage networks by sharing the arc on several metal plates.**
In this way we have the double effect of **cooling the arc** and **multiplying the number of anodes and cathodes in series** with the arc itself, so the **total voltage drop of the arc** v_a **increases** because it is equal to the addition of each voltage drop on each cell contained between two metal plates of the interruption chamber. The switches constructed in this way are called **long arc circuit breakers**.



Short-circuit current interruption

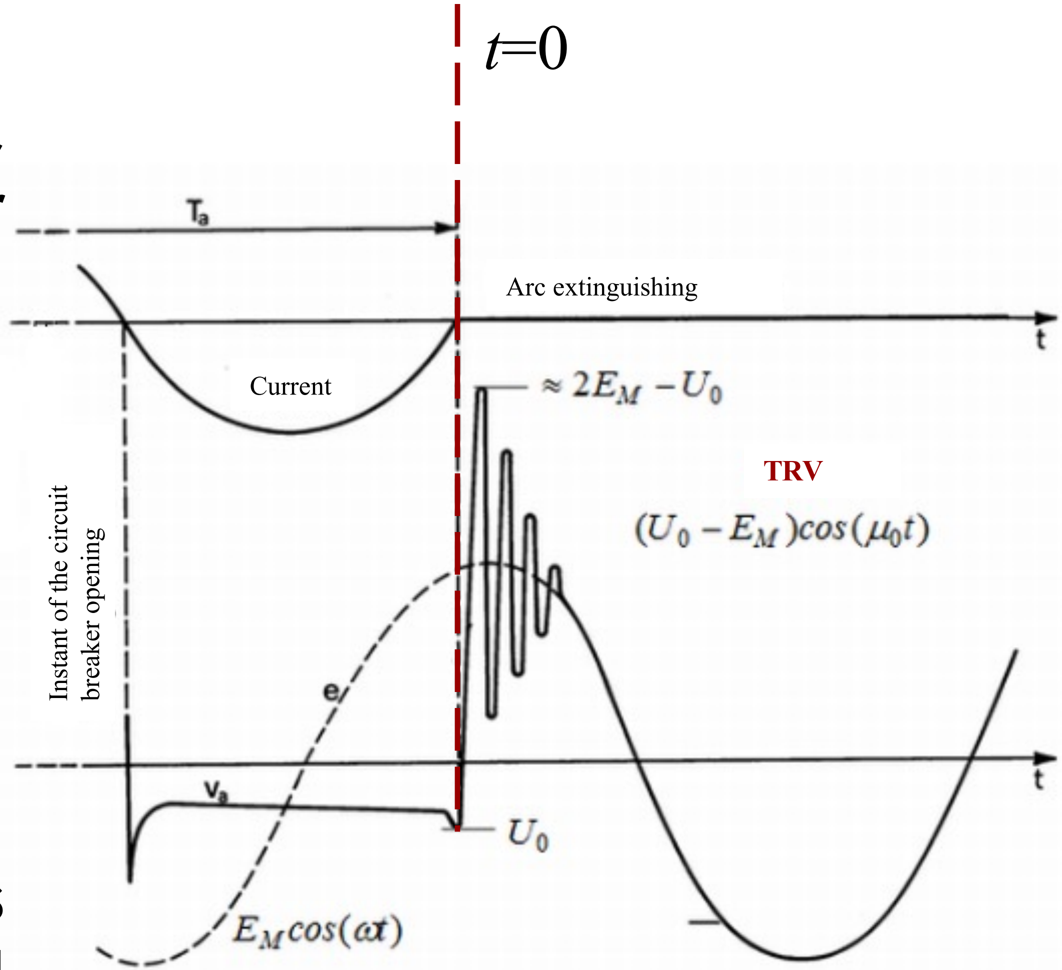
In the case of electrical networks at **medium- and high-voltage**, the nominal voltage $e(t)$ is **much greater than the arc voltage**. In this case, **short arc circuit breakers** are used.

The principle used for interrupting the arc is based on restoring the insulation conditions between the circuit breaker contacts after the current passes zero without allowing the arc to reignite.



Short-circuit current interruption

For this circuit breaker topology, we have $v_a << e$, and **as soon as the power factor in short-circuit is approximately equal to zero** (the resistive component of the equivalent circuit for the short-circuit is negligible), **at the instant when the short-circuit current passes zero and the arc is extinguished, the nominal voltage $e(t)$ has its maximum value E_M.**



Short-circuit current interruption

The differential equation of the circuit where the circuit breaker is placed is as follows:

$$E_M cos(\omega t) = RC\frac{du(t)}{dt} + LC\frac{d^2u(t)}{dt^2} + u(t)$$

If $\mu_0 = \frac{1}{\sqrt{LC}}$ and $R \approx 0$:

$$\mu_0^2 E_M cos(\omega t) = \frac{d^2u(t)}{dt^2} + \mu_0^2 u(t)$$

The solution is as follows:

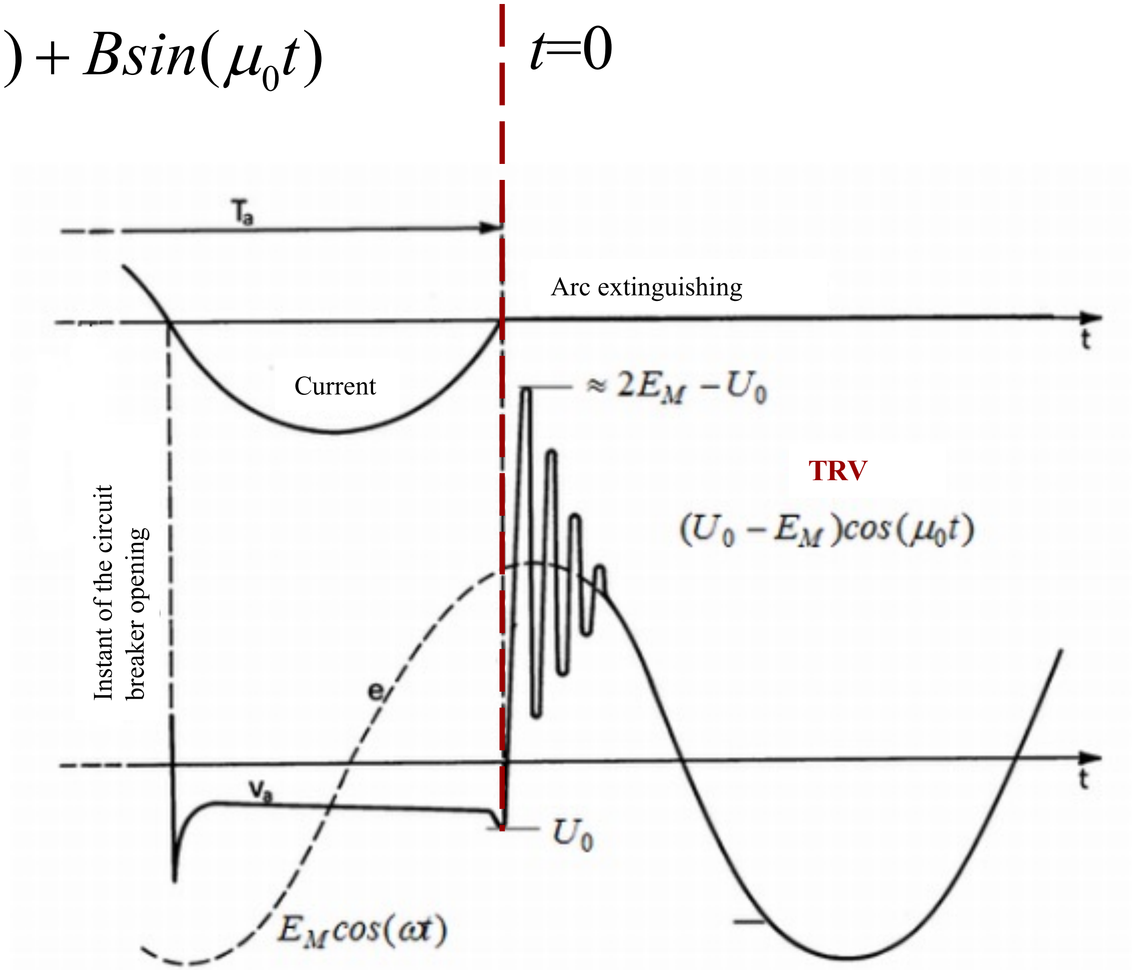
$$u(t) = E_M cos(\omega t) + Acos(\mu_0 t) + Bsin(\mu_0 t)$$

Short-circuit current interruption

$$u(t) = E_M cos(\omega t) + Acos(\mu_0 t) + Bsin(\mu_0 t)$$

For t=0 $u(0)$=U_0

U_0 is the arc voltage at t=0 and $U_0 << E_M$



Therefore:

$$u(0) = U_0 =$$

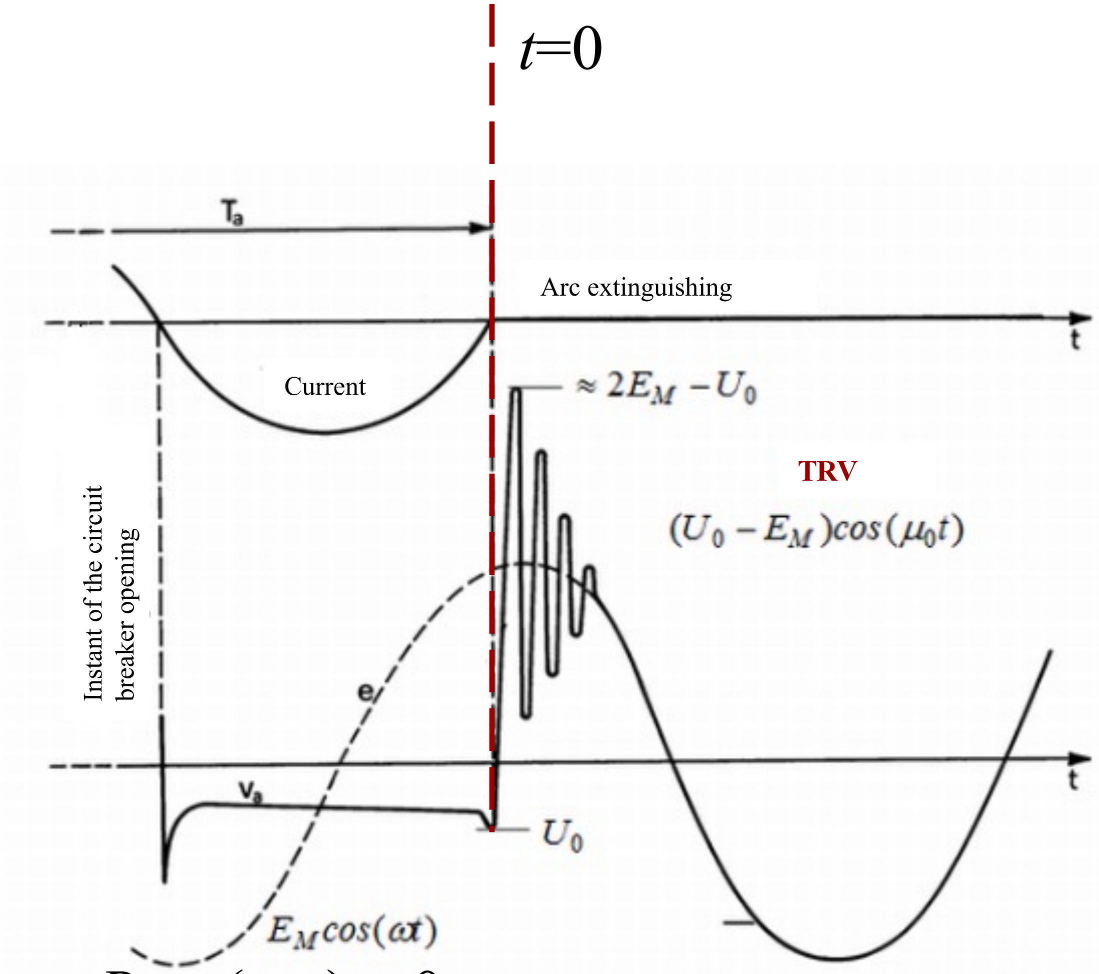
$$= E_M cos(\omega t) + Acos(\mu_0 t) + Bsin(\mu_0 t) =$$

$$= E_M + A$$

Short-circuit current interruption

Additionally, for t=0

$$i(0) = C\frac{du(0)}{dt}\bigg|_{t=0} = 0$$



Therefore:

$$\frac{du(0)}{dt}\bigg|_{t=0} = 0 =$$

$$-\omega E_M sin(\omega t) - \mu_0 A sin(\mu_0 t) + \mu_0 B cos(\mu_0 t) = 0$$

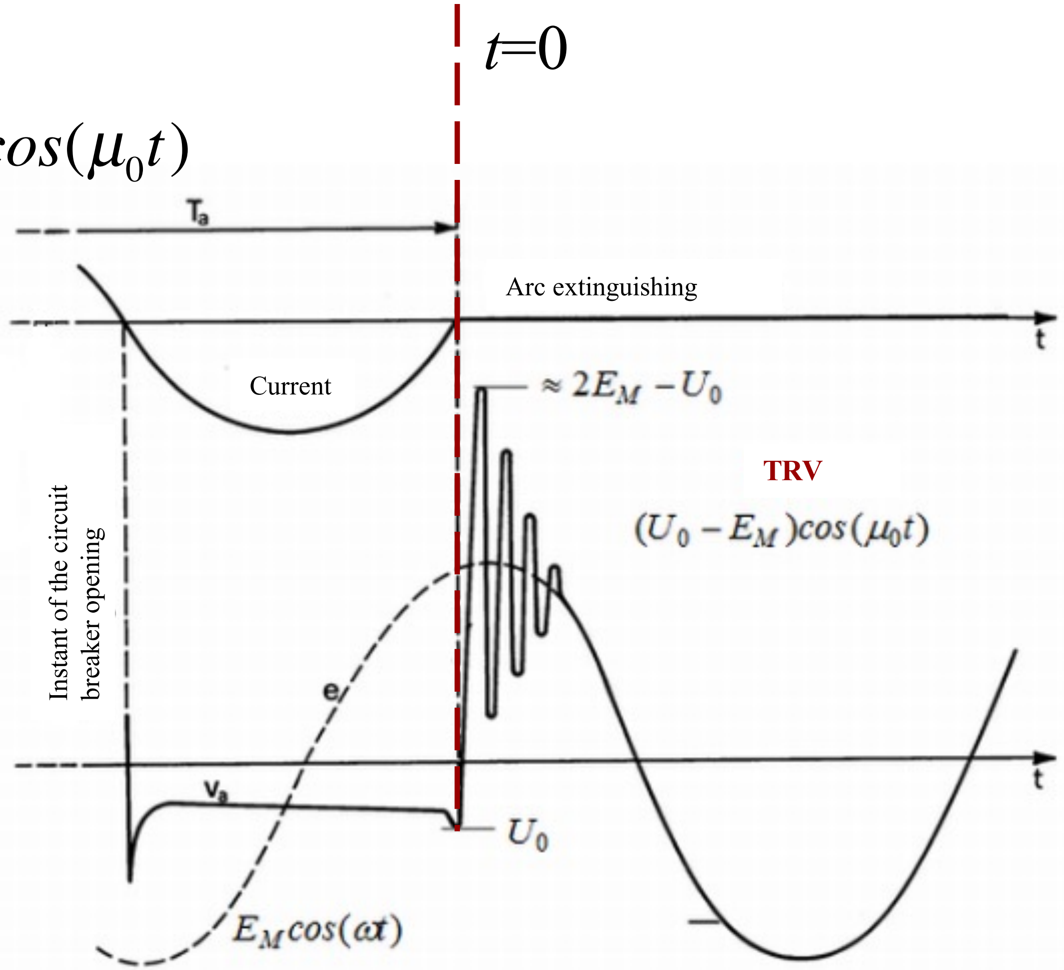
$$B = 0$$

Short-circuit current interruption

The solution is

$$u(t) = E_M cos(\omega t) + (U_0 - E_M) cos(\mu_0 t)$$

After the extinguishing time of the arc π/μ_0 (very short, on the order of ms) there is a **voltage** across the circuit breaker contacts **whose value is approximately equal to** $2E_M$.

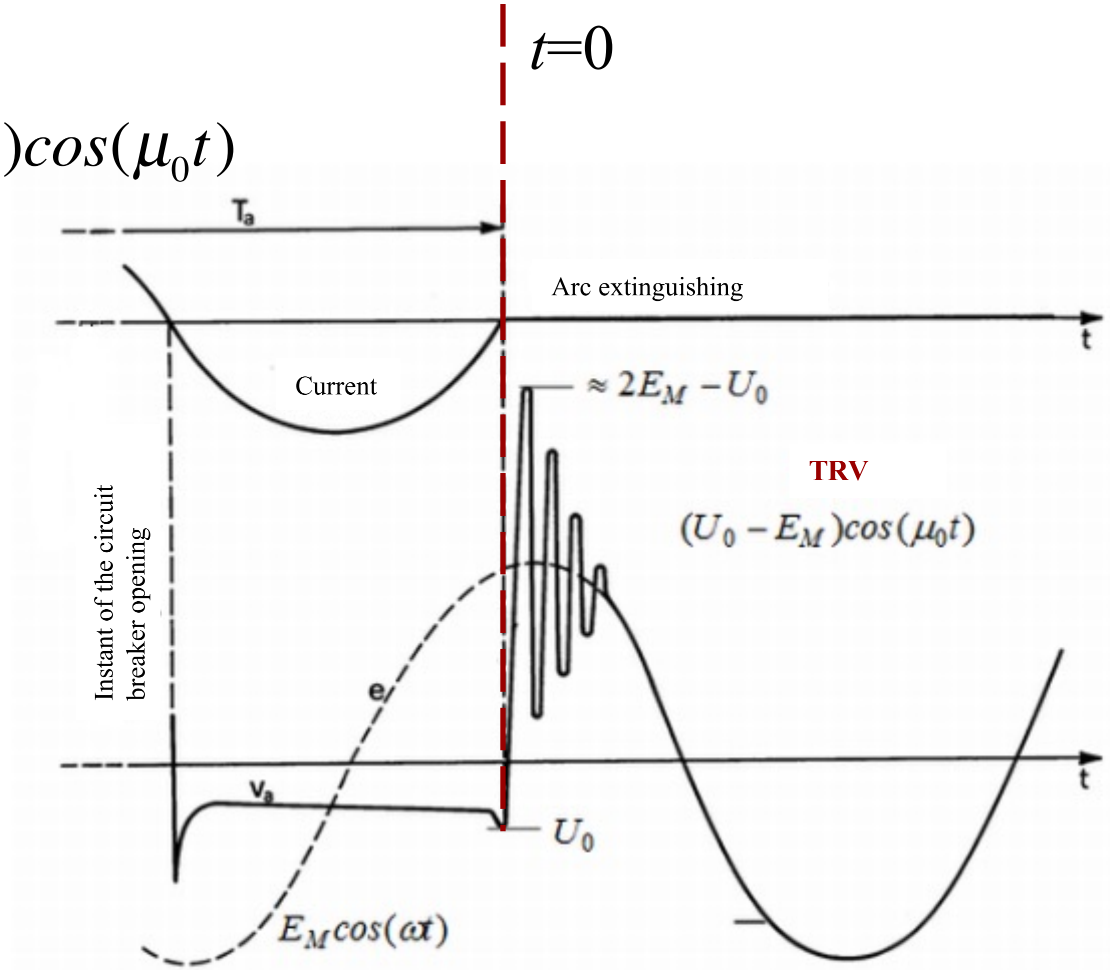


Short-circuit current interruption

The solution is

$$u(t) = E_M cos(\omega t) + (U_0 - E_M) cos(\mu_0 t)$$

The voltage across the circuit breaker contacts just after the circuit breaker opening is called the **transient recovery voltage (TRV).**



Short-circuit current interruption

La solution est

$$u(t) = E_M cos(\omega t) + (U_0 - E_M)cos(\mu_0 t)$$

If we consider a circuit breaker on a line with a nominal voltage equal to 15 kV, i.e., a phase voltage of 8.75 kV, the TRV will be equal to 24.5 kV.